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A separated flow in mixed convection

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A numerical solution is presented for the flow of a uniform stream past a semi-infinite heated flat plate at whose surface the heat flux remains constant. The buoyancy forces oppose the free-stream motion and separation occurs. An examination of the singularities in the skin-friction and heat-transfer coefficients suggests, rather surprisingly, a behaviour as $(\xi_s - \xi)^{\frac{3}{5}}$ at separation.

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1. Introduction

In a numerical computation of the flow against the pressure gradient associated with a linearly decreasing mainstream velocity distribution, Howarth (1938) first noticed the irregular behaviour of an incompressible boundary layer near a point x_s of zero skin friction. Further numerical computations by Hartree (1939) confirmed Howarth's results and stimulated an investigation of the behaviour of the flow in this neighbourhood by Goldstein (1948). Purely on the basis of this numerical evidence Goldstein revoked his tentative analysis of 1930, which suggested an $(x_s - x)^{\frac{1}{4}}$ behaviour of skin friction, in favour of an investigation in which it was assumed that the first condition for the absence of singularities was satisfied. Proceeding from this standpoint Goldstein was able to develop a formal expansion for the stream function about the point of zero skin friction. This included non-integral powers of $x_s - x$ whose coefficients were complicated functions of $\eta \equiv y/(x_s - x)^{\frac{1}{4}}$, where y measured distance normal to the wall. In particular, the expansion forecast a structure close to the wall which resulted in a representation of skin friction which vanished as $(x_s - x)^{\frac{1}{2}}$ in accordance with the numerical evidence. Anomalies associated with exponential decay were settled by Stewartson (1958), who modified the Goldstein expansion to include logarithmic terms whose presence obviated the need to satisfy certain integral conditions inherent in the Goldstein solution. The existence of, and structure at, the singularity is regarded as fully established.

More recently discussions have centred on the behaviour of a laminar compressible boundary layer near a point of zero skin friction. Stewartson (1962) introduced the subject and following an analysis closely patterned on his earlier work on the incompressible case concluded that a general compressible laminar boundary layer can develop a singularity at a point of zero skin friction only if the heat transfer at that point is also zero. This surprising conjecture found support in the computations of Poots (1960) but was thrown in doubt when Williams (1965, private communication) produced numerical evidence of non-zero heat transfer simultaneous with singular behaviour of the skin friction. Further numerical evidence of Merkin (1969) on the analogous problem of convection about a uniform-temperature semi-infinite plate in a parallel uniform stream strengthened the conviction that a more subtle structure, hinted at by Brown & Stewartson (1969), would be required to settle the controversy. Buckmaster (1970, 1971) has pursued this possibility by treating the phenomenon as a parameter perturbation problem following Kaplun's (1967, chap. 3) analysis of the incompressible case. This approach has led him to distinguish between the cold- and hot-wall cases. In the former case a structure involving the introduction of an infinity of new logarithmic-based terms into the Goldstein–Stewartson expansions leads to results which, in the main, agree with Merkin's numerical integration and which hence contradict the Stewartson conjecture. However, it is interesting to note that close to separation the numerical and analytic results are still at variance. For the hot-wall case Buckmaster's investigations support the Stewartson conjecture and again suggest regular behaviour when the heat transfer is non-zero. This conclusion remains in disaccord with Williams's numerical evidence.

Most certainly then, this problem provides a classical example of mutually beneficial interaction between analytic and numerical investigations. As Buckmaster points out, however, accurate numerical integrations to separation are scarce. It was indeed fortuitous that Merkin's work proved to be precisely mathematically analogous to the situation originally described by Stewartson. Also of interest in this work is the separation phenomenon in a context which is not completely governed by a mainstream pressure gradient but rather by the interaction of a uniform stream and retarding buoyancy forces. It is now possible to exploit this phenomenon in mixed convection to consider flow to separation in further circumstances amenable to accurate numerical integration. A circumstance which particularly commends itself is the amendment of the uniform-temperature constraint for a plate to that of uniform heat flux. It is this situation that this paper seeks to examine. It is anticipated that such numerical information will provide the basis for a further investigation into the structure of the irregularities at a point of zero skin friction. Any information, additional to the limited body of literature on this intriguing question of separation within the context of boundary-layer equations, would seem to be welcome at the present time.

2. The problem

A uniform stream U flows along a semi-infinite flat plate extending vertically downwards with its leading edge horizontal. Heat is supplied to the flow by diffusion and convection from the plate as a result of a uniform heat flux q from the surface. This heating, relative to the surrounding ambient temperature T_0, gives rise to buoyant body forces which oppose the free stream. It is anticipated that, near the leading edge, there is little opportuinty for heat from the plate to be taken into the fluid and that consequently the boundary layer is formed chiefly as a result of retardation of the free stream. As the boundary layer develops the effect of buoyancy forces increases until eventually separation occurs.

If it is assumed that

$$U^2/a^2 \ll \Delta T/T_0 \ll 1,$$

where a is the velocity of sound in the fluid and $\Delta T = T_w - T_0$, with T_w the local temperature at the plate, heating due to viscous dissipation can be neglected and the fluid considered incompressible, so that changes in density are significant only in producing buoyancy forces. The kinematic viscosity ν and the thermometric conductivity κ can then be taken as constant and the boundary-layer equations become

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} = -g\beta(T-T_0)+\nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y} = \kappa\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

Here u and v are velocity components associated with increasing x and y respectively, where x measures distance along the plate from the leading edge $x = 0$ and y is measured normally outwards from the plate. T is the temperature of the fluid, g is the acceleration due to gravity and β is the coefficient of thermal expansion. Equations (1)–(3) are to be solved subject to the boundary conditions

$$\left.\begin{aligned} &u = v = 0, \quad \frac{\partial T}{\partial y} = -\frac{q}{k} \quad \text{on} \quad y = 0, \\ &u \to U, \quad T \to T_0 \quad \text{as} \quad y \to \infty, \\ &u = U, \quad T = T_0 \quad \text{at} \quad x = 0, \end{aligned}\right\} \tag{4}$$

where k is the thermal conductivity.

3. Transformations

The non-dimensional parameters governing the flow comprise the local Reynolds number $Re = Ux/\nu$, Grashof number $Gr = g\beta\Delta T x^3/\nu^2$ and Nusselt number $Nu = qx/k\Delta T$. A dimensional analysis of (1)–(3) is instrumental in obtaining a non-dimensional characteristic distance variable

$$\xi = \left(\frac{2^3 Gr^2 Nu^2}{5^2 Re^5}\right)^{\frac{1}{3}} = \left(\frac{2^3 g^2\beta^2 q^2 \nu}{5^2 k^2 U^5}\right)^{\frac{1}{3}} x, \tag{5}$$

which appropriately reflects the local relative importance of viscous and buoyancy forces. Near the leading edge viscous forces dominate and the boundary layer is formed mainly by the retardation of the free stream U. This leads naturally to the following transformations:

$$\Psi = (2\nu Ux)^{\frac{1}{2}} f(\xi,\eta),$$

$$T - T_0 = -\frac{q}{k}\left(\frac{2\nu x}{U}\right)^{\frac{1}{2}} \theta(\xi,\eta),$$

where Ψ is the stream function, $\eta = y(U/2\nu x)^{\frac{1}{2}}$ and ξ is as in (5) above.

As a result of these transformations the boundary-layer equations now read

$$\frac{\partial^3 f}{\partial \eta^3}+f\frac{\partial^2 f}{\partial \eta^2}+5\xi^{\frac{4}{5}}\theta+2\xi\left(\frac{\partial^2 f}{\partial \eta^2}\frac{\partial f}{\partial \xi}-\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial \xi\,\partial \eta}\right)=0, \tag{6}$$

$$\frac{1}{\sigma}\frac{\partial^2\theta}{\partial \eta^2}+f\frac{\partial \theta}{\partial \eta}-\theta\frac{\partial f}{\partial \eta}+2\xi\left(\frac{\partial \theta}{\partial \eta}\frac{\partial f}{\partial \xi}-\frac{\partial f}{\partial \eta}\frac{\partial \theta}{\partial \xi}\right)=0, \tag{7}$$

with boundary conditions

$$\left.\begin{aligned} f=\partial f/\partial\eta=0;\quad \partial\theta/\partial\eta=1\quad &\text{on}\quad \eta=0,\\ \partial f/\partial\eta\to 1,\quad \theta\to 0\quad &\text{as}\quad \eta\to\infty, \end{aligned}\right\} \tag{8}$$

where $\sigma=\nu/\kappa$ is the Prandtl number.

Equations (6) and (7) now provide the basis for a step-by-step numerical solution of the full boundary-layer equations.

4. Numerical solution

In examining the flow associated with a retarded mainstream Terrill (1960) presented a method of solution incorporating the ideas of Hartree & Womersley (1937) and Leigh (1955). By replacing derivatives in the ξ direction by differences and all other quantities by averages the method exploits the parabolic nature of the equations and seeks to establish a velocity profile at a station ξ_2 downstream of a station ξ_1 at which the velocity profile is known. A step-by-step solution ensues whose accuracy is limited only by the time and space required to perform the calculations on the computer.

This method is adapted to deal with the present circumstances, which requires the inclusion of a temperature distribution as well as that of velocity. With $q=\partial f/\partial\eta$, and q_1, θ_1, q_2 and θ_2 the values of velocity and temperature at stations ξ_1 and ξ_2 respectively, the average quantities

$$u=\theta_1+\theta_2,\quad v=q_1+q_2$$

are introduced. If ξ derivatives are replaced by differences and the representations $u^{(m)}$ and $v^{(m)}$ used for the mth iterative approximations to u and v these definitions lead to the following equations for $u^{(m+1)}$ and $v^{(m+1)}$:

$$\frac{d^2v^{(m+1)}}{d\eta^2}+\frac{dv^{(m)}}{d\eta}\int_0^\eta\left[\tfrac{1}{2}v^{(m+1)}+\lambda(v^{(m+1)}-2q_1)\right]d\eta$$
$$+5\left(\frac{\xi_1+\xi_2}{2}\right)^{\frac{4}{5}}u^{(m)}-\lambda v^{(m)}(v^{(m+1)}-2q_1)=0, \tag{9}$$

$$\frac{1}{\sigma}\frac{d^2u^{(m+1)}}{d\eta^2}+\frac{du^{(m+1)}}{d\eta}\int_0^\eta\left[\tfrac{1}{2}v^{(m+1)}+\lambda(v^{(m+1)}-2q_1)\right]d\eta$$
$$-\tfrac{1}{2}u^{(m+1)}v^{(m+1)}-\lambda v^{(m+1)}(u^{(m+1)}-2\theta_1)=0, \tag{10}$$

where $\lambda=(\xi_2+\xi_1)/(\xi_2-\xi_1)$. The iterative procedure of solution requires calculation of $v^{(m+1)}$ from (9) prior to its use in (10). This procedure is found to converge readily.

Equations (9) and (10) are now solved by introducing finite differences in the η direction. As a result the problem reduces to the solution of two matrix equations of the form

$$\mathbf{A}^{(m)}\mathbf{v}^{(m+1)} = \mathbf{c}^{(m)}, \quad \mathbf{B}^{(m)}\mathbf{u}^{(m+1)} = \mathbf{d}^{(m)}, \tag{11), (12}$$

where the elements of the column vectors $\mathbf{c}^{(m)}$ and $\mathbf{d}^{(m)}$ are all known and specifically incorporate boundary conditions at the wall and infinity (understood to be some suitably large value of η). $\mathbf{A}^{(m)}$ proves to be the same matrix as that given by Terrill (1960) and $\mathbf{B}^{(m)}$ is a band matrix as in Merkin (1969). Accordingly (11) and (12) are solved using the method of Choleski, following these previous authors. The recovery of the velocity and temperature distributions q_2 and θ_2 at ξ_2 is now a formality. It remains to initiate the integration.

The initial profiles are taken as those similarity solutions of the reduced form of (6) and (7) obtained after setting $\xi = 0$ and suppressing ξ derivatives. Since the iteration process fails to converge at $\xi = 0$ the integration is begun at

$$\xi_1 = 5 \times 10^{-6},$$

with an initial step length of 5×10^{-6}. Subsequent step lengths are duly enlarged when the maximum number of iterations needed in going from ξ_1 to ξ_2 is less than 4.

Errors arise from using finite differences in both the ξ and η directions. The size of truncation errors in the η direction can be checked using finite-difference estimates whilst errors in the ξ direction are controlled by prescribing a maximum modulus of deviation between a one-step and two-step solution between stations ξ_1 and ξ_2. The values of q_2 and θ_2 obtained from integrating at the half-intervals are the ones used as initial profiles for the next full step of the solution. The level of accuracy achieved is governed solely by the limitations on available storage space. In this instance integrations in the η direction were carried out with $\eta = 0{\cdot}1\ (0{\cdot}1)\ 7{\cdot}2$ and a maximum modulus of deviation of 5×10^{-5}. An overall accuracy of at least three decimal places is therefore anticipated.

5. Flow parameters

The aim of this investigation has been to obtain the point at which separation occurs, understood in the context of this paper to be the point of zero skin friction, and examine the behaviour of the boundary-layer equations at this station as suggested by the numerical evidence. It is desirable therefore to incorporate into the numerical integration program an evaluation of this fundamental flow parameter, given by the skin-friction coefficient

$$\tau_w = \left(\frac{5^2k^2\nu^2}{U^4 2^3 g^2\beta^2 q^2}\right)^{\frac{1}{5}} \left(\frac{\partial u}{\partial y}\right)_0 = (2\xi)^{-\frac{1}{2}}(f_{\eta\eta})_0. \tag{13}$$

Other flow parameters of interest are the heat-transfer coefficient

$$Q = -\left(\frac{5^2k^2\nu^2U^2}{2^3g^2\beta^2q^2}\right)^{\frac{1}{5}} \frac{1}{\Delta T}\left(\frac{\partial T}{\partial y}\right)_0 = -\frac{(2\xi)^{-\frac{1}{2}}}{(\theta)_0}, \tag{14}$$

the momentum thickness

$$\delta_2 = \left(\frac{2^3 g^2 \beta^2 q^2}{5^2 k^2 \nu^2 U^2}\right)^{\frac{1}{5}} \int_0^\infty \frac{u}{U}\left(1-\frac{u}{U}\right) dy = (2\xi)^{\frac{1}{2}} \int_0^\infty f_\eta (1-f_\eta)\, d\eta, \tag{15}$$

and the temperature thickness

$$\delta_T = \left(\frac{2^3 g^2 \beta^2 q^2}{5^2 k^2 \nu^2 U^2}\right)^{\frac{1}{5}} \int_0^\infty \frac{u}{U}\left(\frac{T-T_0}{\Delta T}\right) dy = (2\xi)^{\frac{1}{2}} \int_0^\infty \frac{f_\eta \theta}{(\theta)_0}\, d\eta. \tag{16}$$

On $\eta = 0$
$$\partial^2 q/\partial \eta^2 = -5\xi^{\frac{3}{2}}(\theta)_0, \quad \partial^3 q/\partial \eta^2 = -5\xi^{\frac{3}{2}}$$

and a Taylor series expansion for q, at fixed ξ, yields

$$(\partial q/\partial \eta)_0 = [(16q_1 - q_2 + 30h^2\xi^{\frac{3}{2}}(\theta)_0 + \tfrac{20}{3}h^3\xi^{\frac{3}{2}})/14h] + O(h^4). \tag{17}$$

This representation and the use of the Euler–Maclaurin formula to calculate the integrals in (15) and (16) facilitates the evaluation of the above flow parameters from velocity and temperature profiles at a particular station.

6. Results ($\sigma \equiv 1$)

Initial and separation profiles

Numerical integration is initiated using temperature and velocity distributions $\theta_0(\eta)$ and $f_0'(\eta)$ obtained as solutions of the following fifth-order system of ordinary nonlinear differential equations:

$$f_0''' + f_0 f_0'' = 0, \tag{18}$$

$$\theta_0'' + f_0 \theta_0' - \theta_0 f_0' = 0, \tag{19}$$

where a prime indicates $d/d\eta$, subject to boundary conditions

$$\left.\begin{aligned} f_0 = f_0' = 0; \quad \theta_0' = 1 \quad \text{on} \quad \eta = 0, \\ f_0' \to 1; \quad \theta_0 \to 0 \quad \text{as} \quad \eta \to \infty, \end{aligned}\right\} \tag{20}$$

Solution of this two-point boundary-value problem is accomplished once values of $f_0''(0)$ and $\theta_0(0)$ have been established which lead to the correct behaviour at infinity when (18) and (19) are integrated outwards from $\eta = 0$. These values are

$$f_0''(0) = 0{\cdot}4696, \quad \theta_0(0) = -1{\cdot}5406.$$

Separation occurs when $\xi = \xi_s = 0{\cdot}141955$,

at which station $f_0''(0) = 0; \quad \theta_0(0) = -1{\cdot}9731.$

Initial and separation velocity and temperature profiles are plotted in figure 1. Numerical tabulations of intermediate profiles are available from the author.

Numerical results

Table 1 gives the values of flow parameters at various ξ up to, and including, separation. Figures 2(a) and (b) indicate graphically the behaviour of both the skin-friction coefficient and the heat-transfer coefficient in the vicinity of separation. From them it is seen that

$$\tau_w \to 0, \quad Q \to 0{\cdot}951, \quad \frac{d\tau_w}{d\xi} \to \infty, \quad \frac{dQ}{d\xi} \to \infty \quad \text{as} \quad \xi \to \xi_s.$$

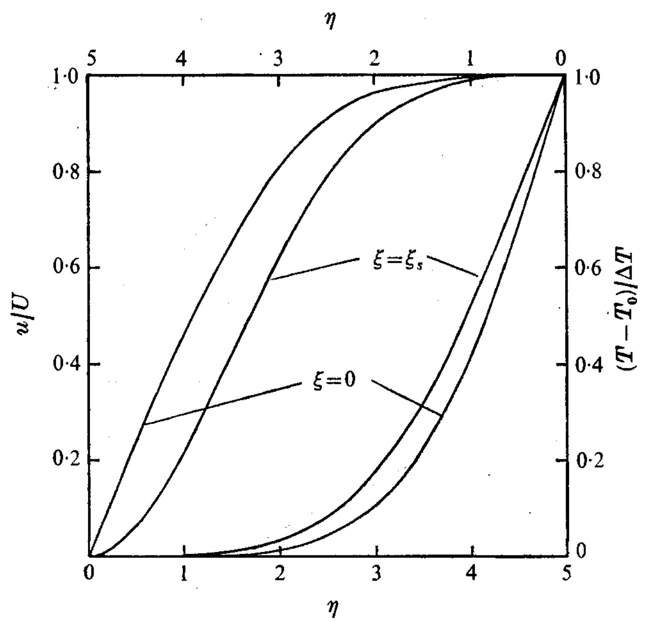

FIGURE 1

ξ	τ_w	Q	δ_2	δ_T
0·00002	74·2496	102·5784	0·0030	0·0021
0·00032	18·5652	25·6453	0·0119	0·0082
0·00128	9·2786	12·8217	0·0237	0·0164
0·00448	4·9457	6·8502	0·0444	0·0307
0·00608	4·2372	5·8783	0·0518	0·0357
0·00768	3·7617	5·2284	0·0582	0·0401
0·01760	2·4389	3·4436	0·0883	0·0606
0·03010	1·8026	2·6193	0·1159	0·0788
0·04260	1·4487	2·1867	0·1385	0·0931
0·05510	1·2035	1·9065	0·1583	0·1050
0·06000	1·1239	1·8200	0·1656	0·1092
0·08000	0·8521	1·5468	0·1931	0·1237
0·09000	0·7353	1·4412	0·2060	0·1296
0·10000	0·6244	1·3480	0·2185	0·1347
0·11000	0·5152	1·2633	0·2306	0·1389
0·12000	0·4025	1·1832	0·2425	0·1419
0·13000	0·2770	1·1023	0·2543	0·1432
0·13500	0·2016	1·0577	0·2601	0·1429
0·13900	0·1240	1·0147	0·2648	0·1409
0·14100	0·0663	0·9845	0·2672	0·1387
0·14150	0·0441	0·9732	0·2677	0·1376
0·14170	0·0320	0·9671	0·2679	0·1369
0·14190	0·0135	0·9579	0·2682	0·1358
0·14193	0·0085	0·9554	0·2682	0·1355
0·14194	0·0062	0·9543	0·2682	0·1353
0·141945	0·0030	0·9527	0·2682	0·1351
0·1419451	0·0025	0·9525	0·2682	0·1351
0·1419452	0·0020	0·9523	0·2682	0·1351
0·1419453	0·0015	0·9520	0·2682	0·1350
0·1419454	0·0008	0·9516	0·2682	0·1350
0·1419455	0	0·9512	0·2682	0·1350

TABLE 1

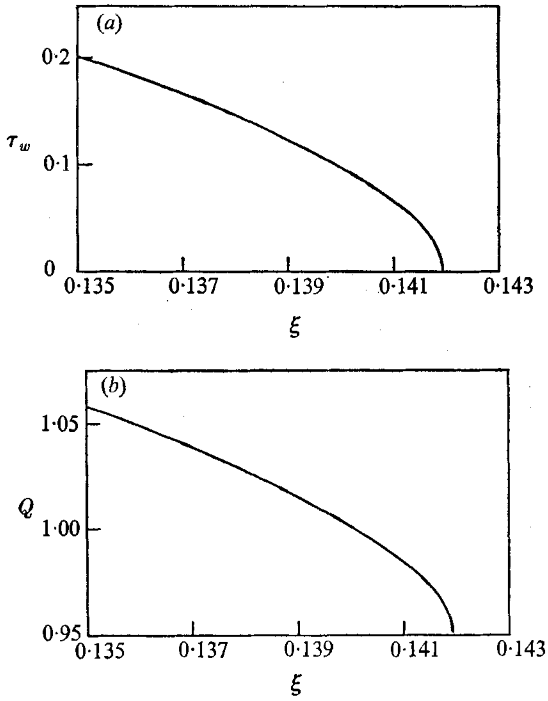

FIGURE 2

Structural evidence

To confirm the validity of his suggested structure in the vicinity of separation Buckmaster analysed Merkin's numerical results for skin friction and heat transfer on a log–log scale. It was clear that both parameters behaved approximately as $(x_s - x)^{\frac{1}{2}}$ except close to separation. This satisfied Buckmaster that his expansion structure was correct. From his analysis the skin friction would vanish as $(x_s - x)^{\frac{1}{2}} \log (x_s - x)$, which, in numerical work, would only be revealed as approximate square-root behaviour. The discrepancy in the immediate vicinity of separation was accredited to errors in Merkin's tabulated results. It is instructive to examine the numerical evidence of this paper in a similar manner and log–log plots of skin-friction and heat-transfer coefficients are presented in figure 3. Bearing in mind that there is some doubt as to the true accuracy of the fourth decimal place in table 1, the evidence nevertheless points overwhelmingly to a behaviour as $(\xi_s - \xi)^{\frac{3}{5}}$ for both skin friction and heat transfer. The discrepancies between this conjecture and the results in the immediate vicinity of separation may again be attributed to accuracy limitations of the numerical solution.

7. Discussion

In his first examination of the boundary-layer equations Goldstein (1930) acknowledged the role of the transformation $\xi = x^{1/n}, n\xi\eta = y, \Psi(x,y) = \xi^{n-1}f(\xi,\eta)$, in terms of which $u = \xi^{n-2}f_\eta/n$. Expanding $f(\xi,\eta)$ as a power series in ξ with

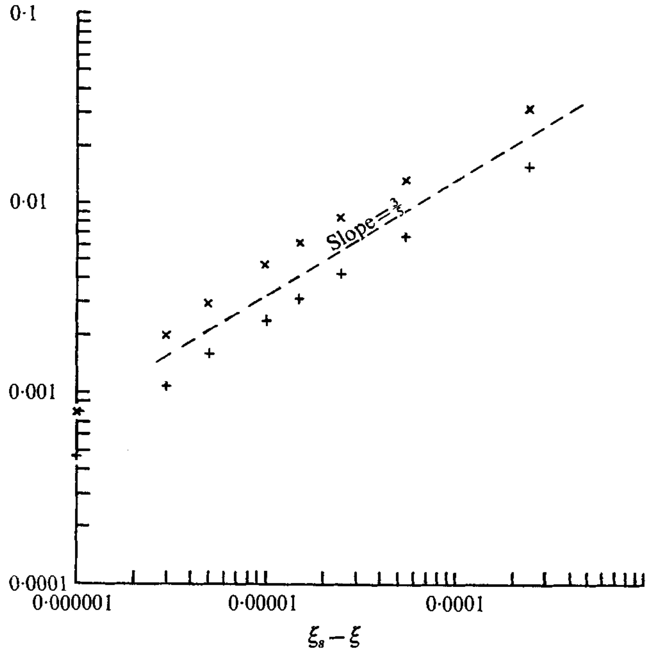

FIGURE 3. Log–log plot. ×, skin friction; +, heat transfer.

coefficients functions of η and considering the limiting behaviour as $\eta \to \infty$ led to an appreciation of the value of n appropriate to extending solution of the boundary-layer equations for given forms of initial profile. In particular, the value $n = 4$ was demonstrated as appropriate to a profile incorporating a double zero at the origin, i.e. a separation profile

$$\begin{aligned} u_0 &= a_2 y^2 + a_3 y^3 + \dots, \\ &= a_2 \xi^2 \eta^2 + a_3 \xi^3 \eta^3 + \dots . \end{aligned}$$

Such transformation in conjunction with series solution appeared to suggest a skin friction vanishing as $(x_s - x)^{\frac{1}{4}}$ since the general solution for the coefficient of ξ^2 allowed a finite contribution at $\eta = 0$. Only under the specification $a_2 = \frac{1}{2}$, coinciding with the fulfilment of the first condition for the absence of singularities, were theory and numerical evidence reconciled. No instances of physical significance were to hand for $n > 4$. Perhaps the contents of this paper provide such an instance. A fifth-root transformation would certainly seem to be required to give any chance of dealing with the skin-friction behaviour of figure 3. The question then arises as to the nature of the separation profile which can fit into this picture and the correlation between its coefficients and those of a skin-friction series solution. Furthermore will one have to resort to a structure of the Buckmaster type to establish consistency between theoretical and numerical results? A theoretical investigation into these questions is at present in progress.

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